Solutions - Homework 1

(Due date: September 20th @ 5:30 pm)

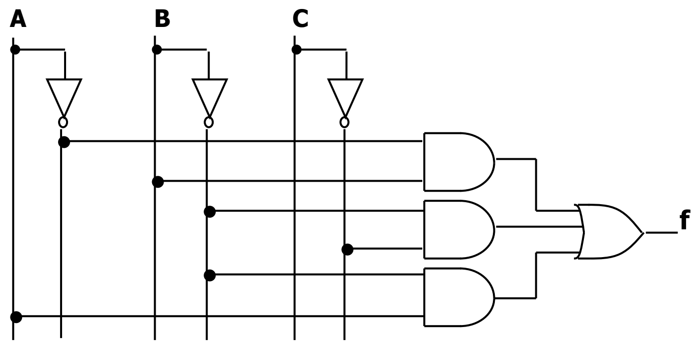
Presentation and clarity are very important!

PROBLEM 1 (27 PTS)

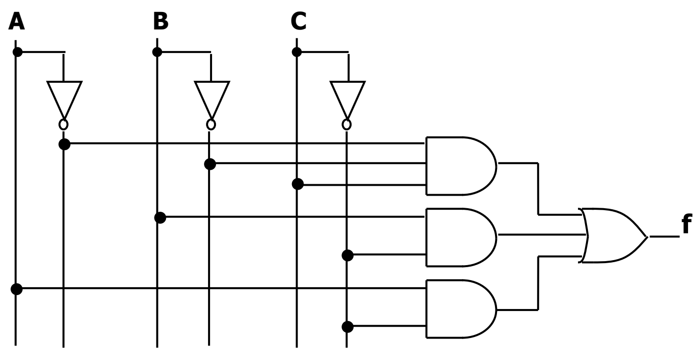
a) Simplify the following functions using ONLY Boolean Algebra Theorems. For each resulting simplified function, sketch the logic circuit using AND, OR, XOR, and NOT gates. (14 pts)

- ✓ $F = \overline{(A \oplus \bar{B})C + AB\bar{C}}$
- ✓ $F = \overline{\overline{XY} + Y(\bar{Z} + \bar{X})}$
- ✓ $F(A,B,C) = \prod(M_0, M_3, M_5, M_7)$
- ✓ $F = (A + \bar{C} + \bar{D})(\bar{B} + \bar{C} + D)(A + \bar{B} + \bar{C})$

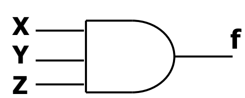
✓ $F = \overline{(A \oplus \bar{B})C + AB\bar{C}} = \overline{(AB + \bar{A}\bar{B})C + AB\bar{C}} = \overline{ABC + \bar{A}\bar{B}C + AB\bar{C}} = \overline{AB + \bar{A}\bar{B}C} = \overline{AB}.\overline{\bar{A}\bar{B}C}$
$= (\bar{A} + \bar{B})(A + B + \bar{C}) = \bar{A}B + \bar{A}\bar{C} + \bar{B}A + \bar{B}\bar{C} = B\bar{A} + \bar{B}\bar{C} + \bar{A}\bar{C} + \bar{B}A = B\bar{A} + \bar{B}\bar{C} + \bar{B}A$



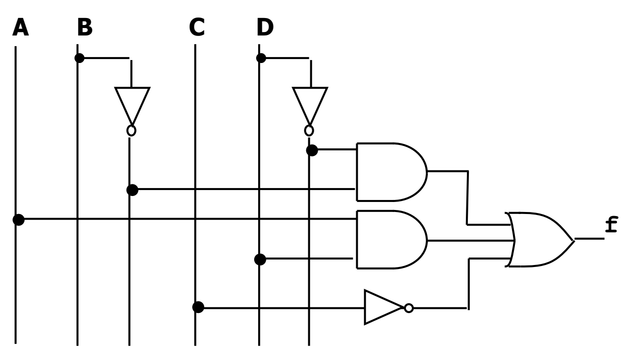
✓ $F(A,B,C) = \prod(M_0, M_3, M_5, M_7) = \sum(m_1, m_2, m_4, m_6) = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C} = \bar{A}\bar{B}C + \bar{C}(\bar{A}B + A\bar{B} + AB)$
$= \bar{A}\bar{B}C + \bar{C}(\bar{A}B + A) = \bar{A}\bar{B}C + \bar{C}(\bar{A} + A)(A + B) = \bar{A}\bar{B}C + \bar{C}A + \bar{C}B$



✓ $F = \overline{Y(\bar{Z} + \bar{X}) + \overline{XY}} = \overline{Y\bar{Z} + Y\bar{X} + \bar{X} + \bar{Y}} = \overline{Y\bar{Z} + \bar{X} + \bar{Y}} = \overline{\bar{X} + (\bar{Y} + Y)(\bar{Y} + \bar{Z})} = \overline{\bar{X} + \bar{Y} + \bar{Z}} = XYZ$



✓ $F = (A + \bar{C} + \bar{D})(\bar{B} + \bar{C} + D)(A + \bar{B} + \bar{C}) = (D + \bar{B} + \bar{C})(\bar{D} + A + \bar{C})(A + \bar{B} + \bar{C}) = (D + \bar{B} + \bar{C})(\bar{D} + A + \bar{C})$
$= (D + \bar{B} + \bar{C})(\bar{D} + A + \bar{C}) = D(A + \bar{C}) + \bar{D}(\bar{B} + \bar{C}) + (A + \bar{C})(\bar{B} + \bar{C}) = D(A + \bar{C}) + \bar{D}(\bar{B} + \bar{C})$
$= \bar{D}\bar{B} + DA + \bar{C}$



b) Using ONLY Boolean Algebra Theorems, demonstrate: (5 pts)

$$X(Y\oplus Z) = (XY)\oplus(XZ)$$

$$(XY)\oplus(XZ) = \overline{XY}(XZ) + XY(\overline{XZ}) = (\bar{X} + \bar{Y})XZ + XY(\bar{X} + \bar{Z}) = \bar{Y}XZ + XY\bar{Z} = X(\bar{Y}Z + Y\bar{Z}) = X(Y\oplus Z)$$

c) For the following Truth table with two outputs: (8 pts)
- Provide the Boolean functions using the Canonical Sum of Products (SOP), and Product of Sums (POS).
- Express the Boolean functions using the minterms and maxterms representations.
- Sketch the logic circuits as Canonical Sum of Products and Product of Sums.

x	y	z	f_1	f_2
0	0	0	1	0
0	0	1	0	1
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Sum of Products

$f_1 = \bar{x}\bar{y}\bar{z} + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$

$f_2 = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$

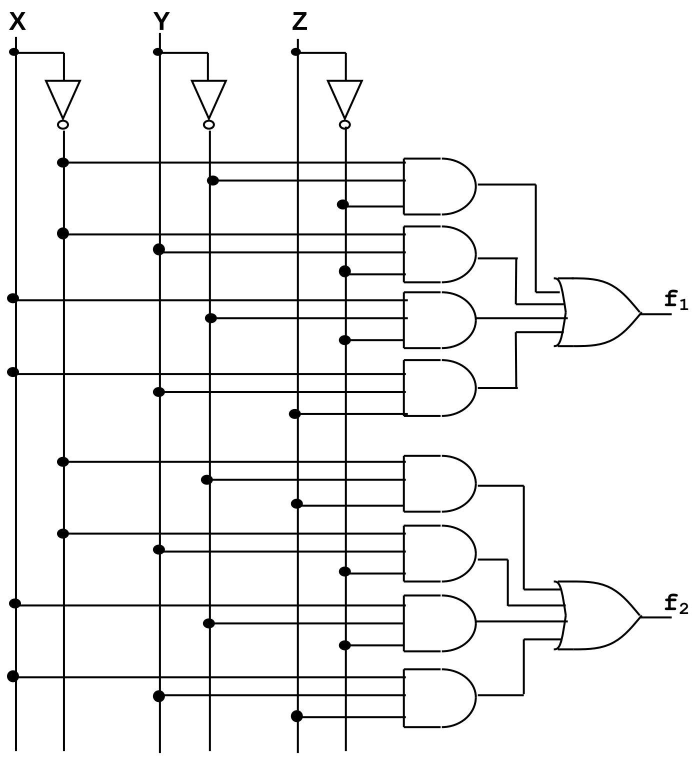
Product of Sums

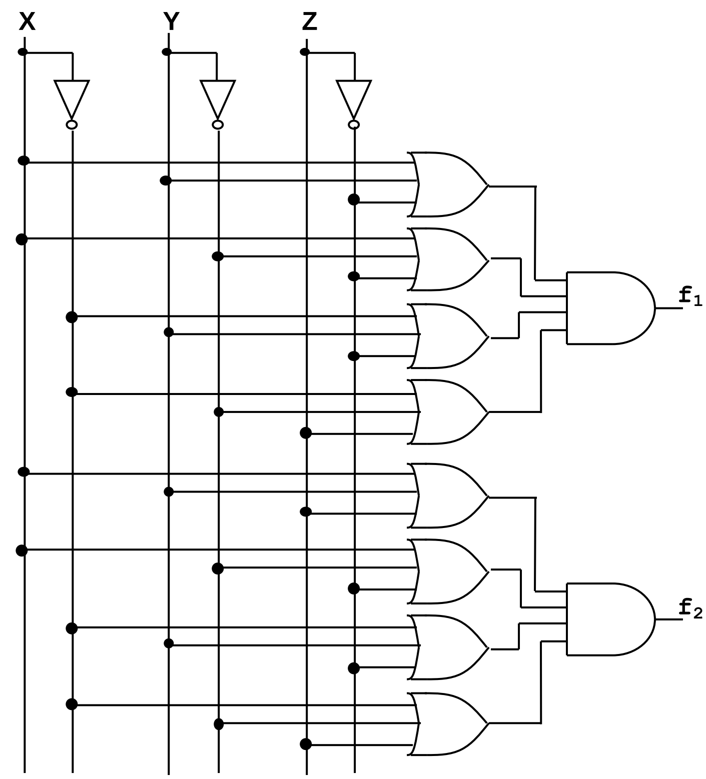
$f_1 = (x + y + \bar{z})(x + \bar{y} + \bar{z})(\bar{x} + y + \bar{z})(\bar{x} + \bar{y} + z)$

$f_2 = (x + y + z)(x + \bar{y} + \bar{z})(\bar{x} + y + \bar{z})(\bar{x} + \bar{y} + z)$

Minterms and maxterms: $f_1 = \sum(m_0, m_2, m_4, m_7) = \prod(M_1, M_3, M_5, M_6)$.

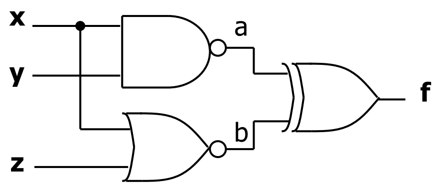
$f_2 = \sum(m_1, m_2, m_4, m_7) = \prod(M_0, M_3, M_5, M_6)$.





PROBLEM 2 (26 PTS)

a) Construct the truth table describing the output of the following circuit and write the simplified Boolean equation (7 pts).

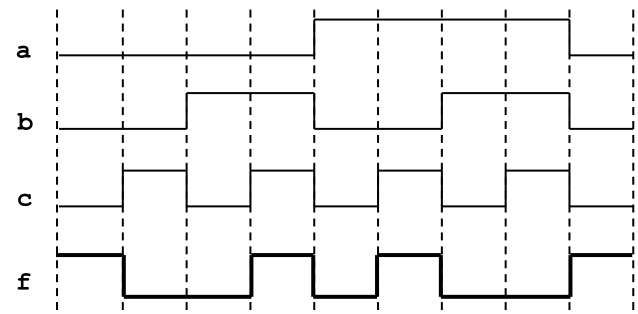


x	y	z	a	b	f
0	0	0	1	1	0
0	0	1	1	0	1
0	1	0	1	1	0
0	1	1	1	0	1
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	0	0	0
1	1	1	0	0	0

$$f = \bar{a}\oplus\bar{b} = a\oplus b = (xy)\oplus(x+z) = (\overline{xy})(x+z) + xy(\overline{x+z}) = (\bar{x}+\bar{y})(x+z) + xy(\bar{x}\bar{z}) = (x+z)(\bar{x}+\bar{y}) = \bar{x}z + x\bar{y}$$

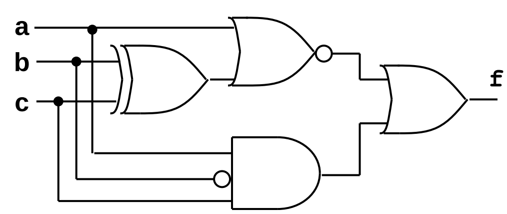
b) The following is the timing diagram of a logic circuit with 3 inputs. Sketch the logic circuit that generates this waveform. Then, complete the VHDL code. (8 pts)

```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
 port ( a, b, c: in std_logic;
        f: out std_logic);
end circ;

architecture st of circ is
  signal x,y: std_logic;
begin
  x <= a and not(b) and c;
  y <= a nor (b xor c);
  f <= x or y;
end st;
```



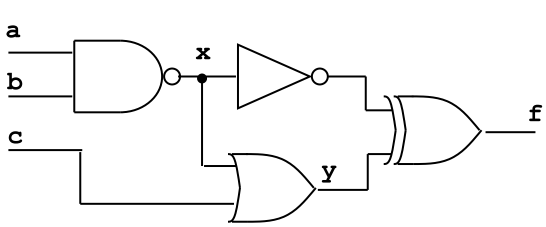
a	b	c	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

c \ ab	00	01	11	10
0	1	0	1	0
1	0	1	0	0

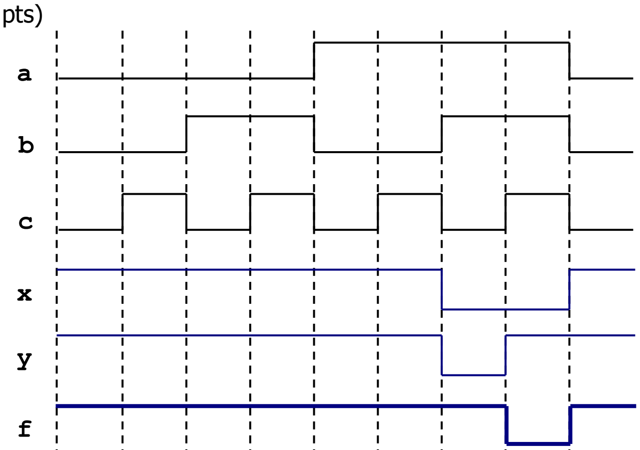


$$f = \bar{a}\bar{b}\bar{c} + \bar{a}bc + a\bar{b}c = \bar{a}\left(\overline{b\oplus c}\right) + a\bar{b}c = \overline{a + (b\oplus c)} + a\bar{b}c$$

c) Complete the timing diagram of the following circuit: (5 pts)

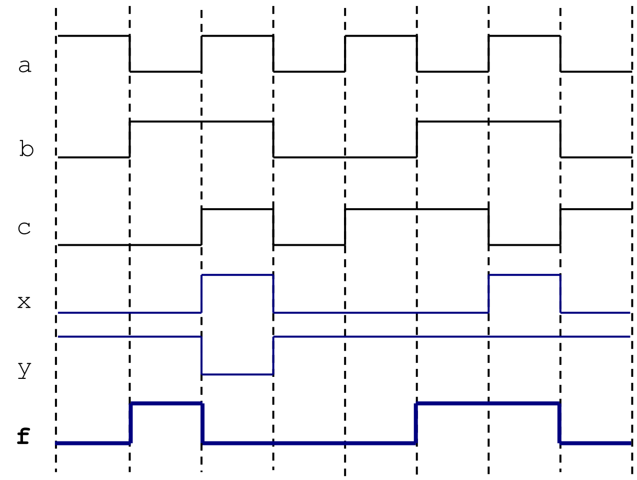


$$f = \bar{x}\oplus y = \overline{x\oplus y}$$



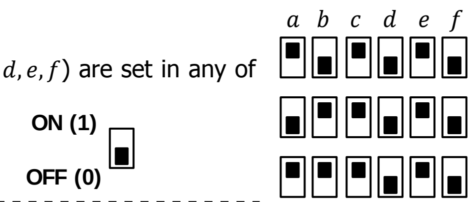
d) Complete the timing diagram of the logic circuit whose VHDL description is shown below: (6 pts)

```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
  port ( a, b, c: in std_logic;
           f: out std_logic);
end circ;

architecture st of circ is
  signal x, y: std_logic;

begin
  y <= x nand c;
  x <= a and b;
  f <= (not y) xor b;
end st;
```

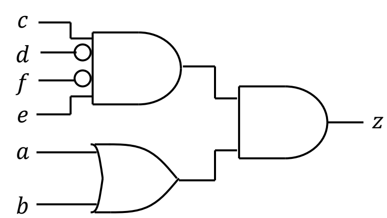


PROBLEM 3 (9 PTS)

- Security combinations: A lock only opens ($z=1$) when the 6 switches (a,b,c,d,e,f) are set in any of the 3 configurations shown in the figure, otherwise the lock is closed ($z=0$).
  - ✓ Provide the Boolean equation for the output z and sketch the logic circuit.



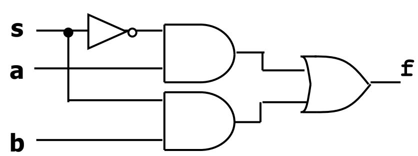
a	b	c	d	e	f	z
1	0	1	0	1	0	1
0	1	1	0	1	0	1
1	1	1	0	1	0	1
All remaining cases						0

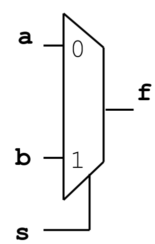


$z = a\bar{b}c\bar{d}e\bar{f} + \bar{a}bc\bar{d}e\bar{f} + abc\bar{d}e\bar{f} = c\bar{d}e\bar{f}(a\bar{b} + \bar{a}b + ab) = c\bar{d}e\bar{f}(a+b)$

PROBLEM 4 (13 PTS)

a) The following circuit has the following logic function: $f = \bar{s}a + sb$.
  - ✓ Complete the truth table of the circuit, and sketch the logic circuit (3 pts)

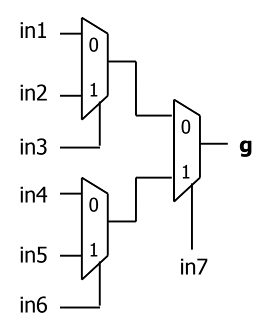




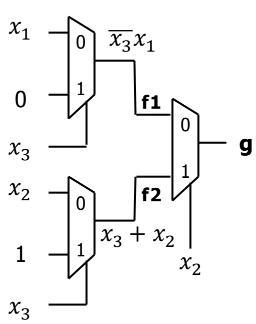
s	a	b	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

b) We can use several instances of the previous circuit to implement different functions. (10 pts)

- For example, the following selection of inputs produce the function: $g = x_2 + x_1\overline{x_3}$. Demonstrate that this is the case.



in1	in2	in3	in4	in5	in6	in7
x_1	0	x_3	x_2	1	x_3	x_2

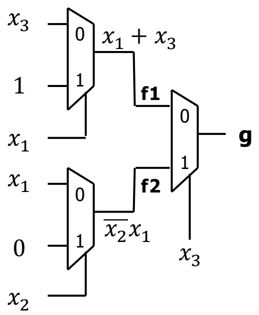


$f_1 = \overline{x_3}(x_1) + x_3(0) = \overline{x_3}x_1$
$f_2 = \overline{x_3}(x_2) + x_3(1) = x_3 + x_2$

$g = \overline{x_2}(\overline{x_3}x_1) + x_2(x_3 + x_2) = \overline{x_3}x_1\overline{x_2} + x_2x_3 + x_2$
$g = \overline{x_3}x_1\overline{x_2} + x_2 = (x_2 + \overline{x_2})(x_2 + \overline{x_3}x_1) = x_2 + x_1\overline{x_3}$

- Given the following inputs, provide the resulting function g (minimize the function).

in1	in2	in3	in4	in5	in6	in7
x_3	1	x_1	x_1	0	x_2	x_3



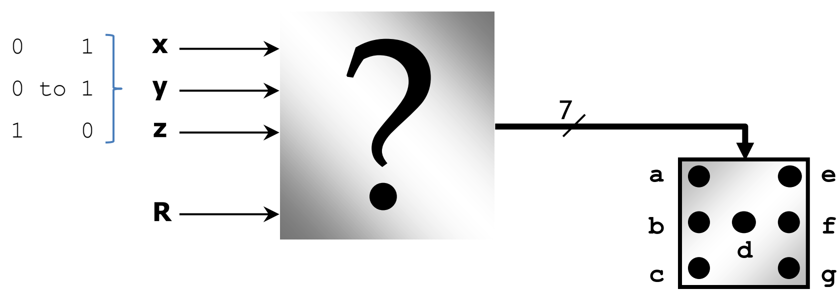
$$f_1 = \overline{x_1}(x_3) + x_1(1) = x_1 + x_3$$
$$f_2 = \overline{x_2}(x_1) + x_2(0) = \overline{x_2}x_1$$

$$g = \overline{x_3}(x_1 + x_3) + x_3(\overline{x_2}x_1) = \overline{x_3}x_1 + x_3\overline{x_2}x_1$$
$$g = x_1(\overline{x_3} + x_3\overline{x_2}) = x_1(\overline{x_3} + x_3)(\overline{x_3} + \overline{x_2}) = x_1\overline{x_3} + x_1\overline{x_2}$$

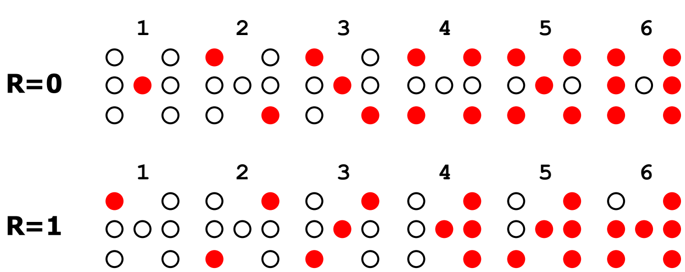
PROBLEM 5 (25 PTS)

- An array of seven LEDs is used to display the results of a roll of a die. Numeric data (1-6) is produced as a 3-bit value. We want to design a logic circuit that converts that 3-bit value to the corresponding 7-bit LED pattern in a die. For example, the code `101` is displayed such that it represents the number '5' in a die side.
- In addition, we have an input `R`. When `R=0`, values are displayed as in a normal die. When `R=1`, values are displayed a little bit different. See figure for details.
- Note: The LEDs are lit with a logical '1' (positive logic). The inputs are active high (or in positive logic).

  - ✓ Complete the truth table for each output (a, b, c, d, e, f, g). Note that it is safe to assume that the inputs x, y, z will not produce the values `000` and `111`.
  - ✓ Provide the simplified expression for each output (a, b, c, d, e, f, g). Use Karnaugh maps for a, b, c, d, e and the Quine-McCluskey algorithm for f, g.



Value	R	x	y	z	a	b	c	d	e	f	g
	0	0	0	0							
1	0	0	0	1							
2	0	0	1	0	1	0	0	0	0	0	1
3	0	0	1	1							
4	0	1	0	0							
5	0	1	0	1							
6	0	1	1	0							
	0	1	1	1							
	1	0	0	0							
1	1	0	0	1							
2	1	0	1	0							
3	1	0	1	1							
4	1	1	0	0							
5	1	1	0	1	0	0	1	1	1	1	1
6	1	1	1	0							
	1	1	1	1							



- To fill out the truth table, note that xyz will not produce the values 000 and 111. Thus, we can use don't care outputs on those instances.

Value	R	x	y	z	a	b	c	d	e	f	g
	0	0	0	0	X	X	X	X	X	X	X
1	0	0	0	1	0	0	0	1	0	0	0
2	0	0	1	0	1	0	0	0	0	0	1
3	0	0	1	1	1	0	0	1	0	0	1
4	0	1	0	0	1	0	1	0	1	0	1
5	0	1	0	1	1	0	1	1	1	0	1
6	0	1	1	0	1	1	1	0	1	1	1
	0	1	1	1	X	X	X	X	X	X	X
	1	0	0	0	X	X	X	X	X	X	X
1	1	0	0	1	1	0	0	0	0	0	0
2	1	0	1	0	0	0	1	0	1	0	0
3	1	0	1	1	0	0	1	1	1	0	0
4	1	1	0	0	0	0	0	1	1	1	1
5	1	1	0	1	0	0	1	1	1	1	1
6	1	1	1	0	0	1	1	1	1	1	1
	1	1	1	1	X	X	X	X	X	X	X

a

yz \ Rx	00	01	11	10
00	X	1	0	X
01	0	1	0	1
11	1	X	X	0
10	1	1	0	0

b

yz \ Rx	00	01	11	10
00	X	0	0	X
01	0	0	0	0
11	0	X	X	0
10	0	1	1	0

c

yz \ Rx	00	01	11	10
00	X	1	0	X
01	0	1	1	0
11	0	X	X	1
10	0	1	1	1

d

yz \ Rx	00	01	11	10
00	X	0	1	X
01	1	1	1	0
11	1	X	X	1
10	0	0	1	0

e

yz \ Rx	00	01	11	10
00	X	1	1	X
01	0	1	1	0
11	0	X	X	1
10	0	1	1	1

$a = R\bar{x}\bar{y} + \bar{R}x + \bar{R}y$
$b = xy$
$c = \bar{R}x + xy + Ry$
$d = Rx + yz + \bar{R}z$
$e = x + Ry$

- $f = \sum m(6,12,13,14) + \sum d(0,7,8,15)$.

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicants
0	m_0 = 0000 ✓	**$m_{0,8}$ = -000**		We can't combine any further, so we stop here
1	m_8 = 1000 ✓	**$m_{8,12}$ = 1-00**		
2	m_6 = 0110 ✓ m_{12}= 1100 ✓	$m_{6,7}$ = 011- ✓ $m_{6,14}$ = -110 ✓ $m_{12,13}$ = 110- ✓ $m_{12,14}$ = 11-0 ✓	**$m_{6,7,14,15}$ = -11-** ~~$m_{7,15,6,14}$ = -11-~~ **$m_{12,13,14,15}$ = 11--** ~~$m_{12,14,13,15}$ = 11--~~	
3	m_7 = 0011 ✓ m_{13}= 1101 ✓ m_{14}= 1110 ✓	$m_{7,15}$ = -111 ✓ $m_{13,15}$ = 11-1 ✓ $m_{14,15}$ = 111- ✓		
4	m_{15}= 1111 ✓			

$f = \bar{x}\bar{y}\bar{z} + R\bar{y}\bar{z} + xy + Rx$

Prime Implicants		Minterms 6	12	13	14
$m_{0,8}$	$\bar{x}\bar{y}\bar{z}$				
$m_{8,12}$	$R\bar{y}\bar{z}$		X		
$m_{6,7,14,15}$	xy	**X**			X
$m_{12,13,14,15}$	Rx		X	**X**	X

$f = xy + Rx$

- $g = \sum m(2,3,4,5,6,12,13,14) + \sum d(0,7,8,15)$.
  Too many minterms. We better optimize: $\bar{g} = \sum m(1,9,10,117) + \sum d(0,7,8,15)$

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicants
0	m_0 = 0000 ✓	$m_{0,1}$ = 000- ✓ $m_{0,8}$ = -000 ✓	**$m_{0,1,8,9}$ = -00-** ~~$m_{0,8,1,9}$ = -00-~~	We can't combine any further, so we stop here
1	m_1 = 0001 ✓ m_8 = 1000 ✓	$m_{1,9}$ = -001 ✓ $m_{8,9}$ = 100- ✓ $m_{8,10}$ = 10-0 ✓	**$m_{8,9,10,11}$ = 10--** ~~$m_{8,10,9,11}$ = 10--~~	
2	m_9 = 1001 ✓ m_{10}= 1010 ✓	$m_{9,11}$ = 10-1 ✓ $m_{10,11}$ = 101- ✓		
3	m_{11}= 1011 ✓	**$m_{11,15}$ = 1-11**		
4	m_{15}= 1111 ✓			

$\bar{g} = Ryz + \bar{x}\bar{y} + R\bar{x}$

Prime Implicants		Minterms			
		1	9	10	11
$m_{11,15}$	Ryz				X
$m_{0,1,8,9}$	$\bar{x}\bar{y}$	**X**	X		
$m_{8,9,10,11}$	$R\bar{x}$		X	**X**	X

$\bar{g} = \bar{x}\bar{y} + R\bar{x} = \bar{x}(R + \bar{y}) \qquad \Rightarrow g = \overline{\bar{x}(R + \bar{y})} = x + \bar{R}y$